Artificial Intelligence
CE-417, Group 1
Computer Eng. Department
Sharif University of Technology

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By Mohammad Hossein Rohban, Ph.D.

Courtesy: Most slides are adopted from CSE-573 (Washington U.), original slides for the textbook, and CS-188 (UC. Berkeley).

Problem Space and Uninformed Search

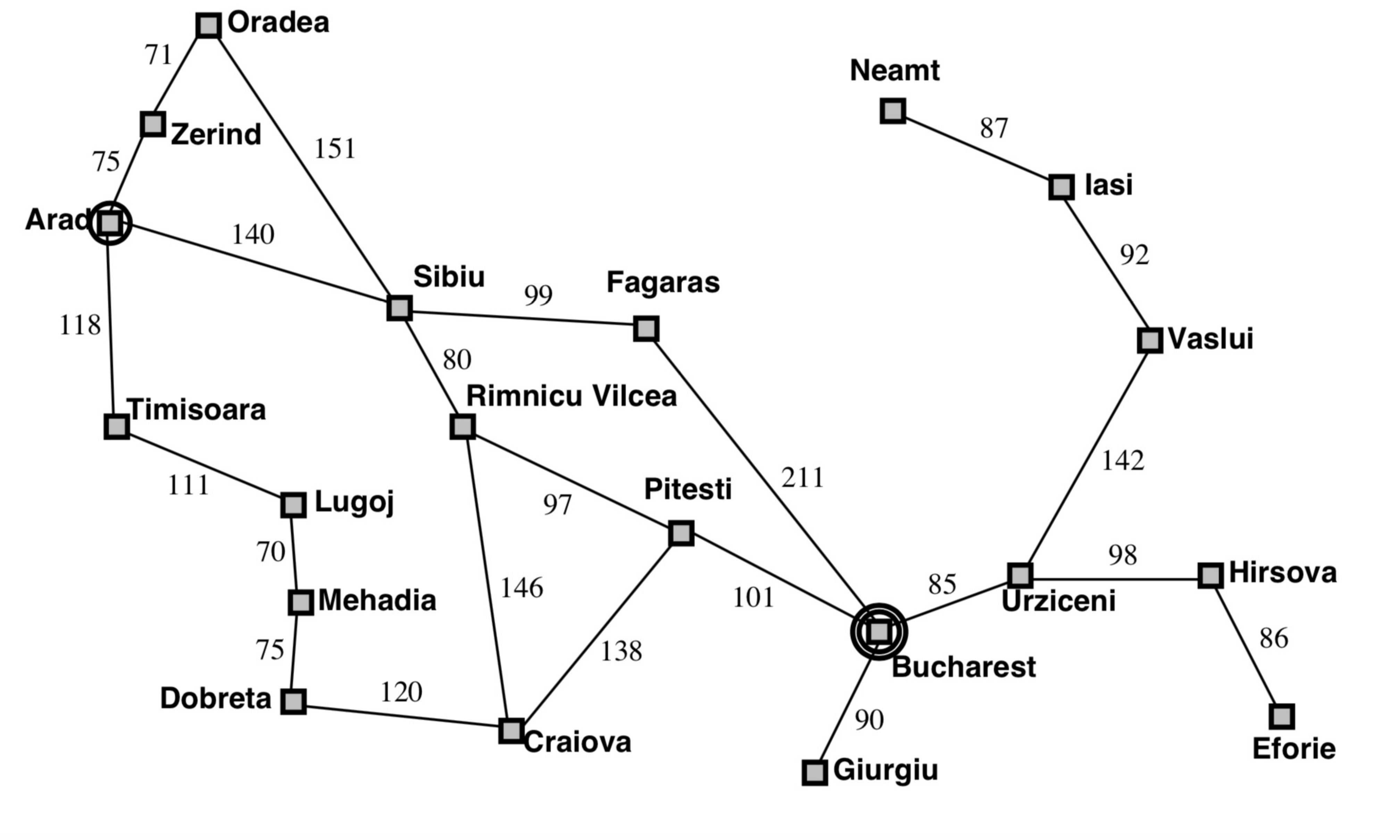
Problem solving agents

- On holiday in Romania; currently in Arad.

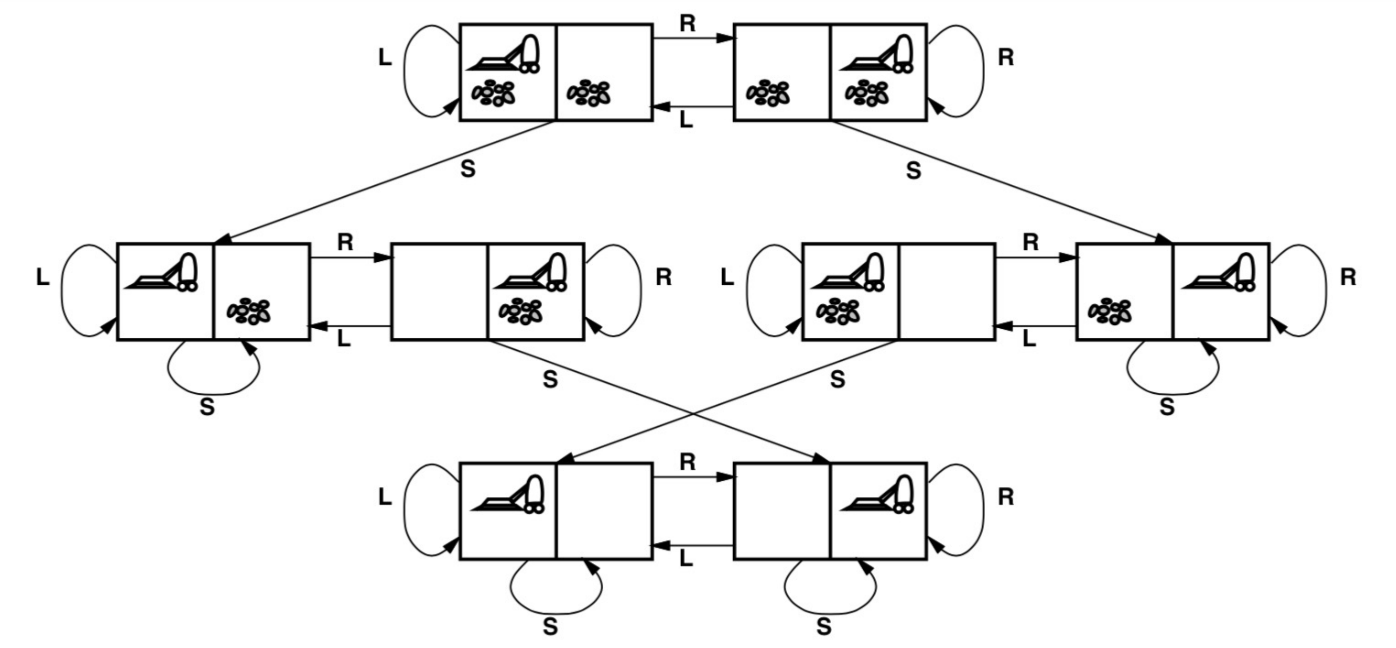
  Flight leaves tomorrow from Bucharest

- Formulate goal: **be in Bucharest**
- Formulate problem:
  - states: **various cities**
  - actions: **drive between cities**
- Find solution: **sequence of cities**, e.g., Arad, Sibiu, Fagaras, Bucharest

Problem solving agents (cont.)

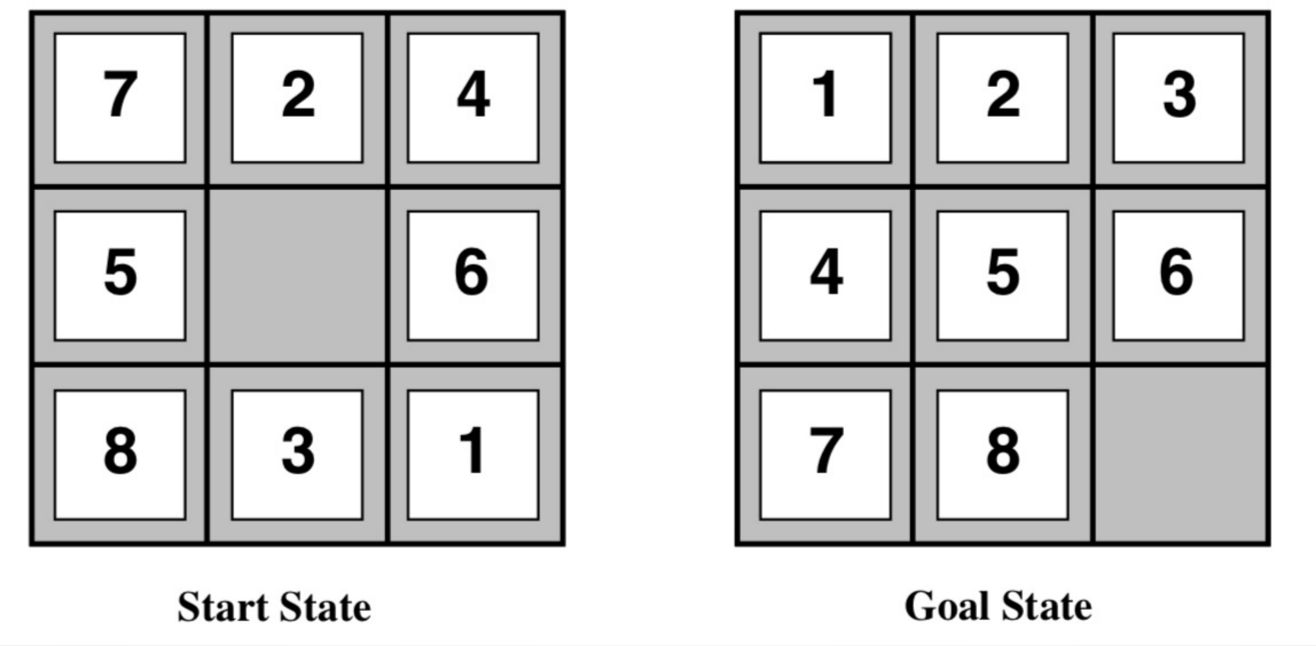


Another example: vacuum world



- States: **integer dirt and robot locations** (ignore dirt amounts etc.)
- Actions: **Left**, **Right**, **Suck**, **NoOp**
- Goal test: **no dirt**
- Path cost: **1 per action (0 for NoOp)**

Another example: The 8-puzzle



- States: **integer locations of tiles** (ignore intermediate positions)
- Actions: **move blank left, right, up, down** (ignore unjamming etc.)
- Goal test: **= goal state** (given)
- Path cost: **1 per move**
- [Note: optimal solution of n-Puzzle family is NP-hard]

Tree Search Algorithms

- Basic idea:

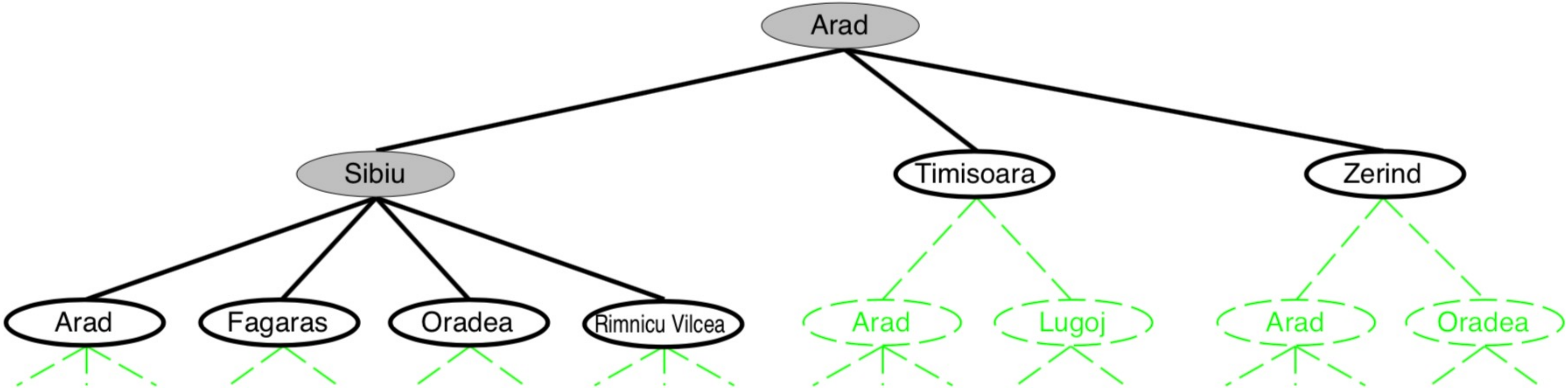
  offline, simulated exploration of state space

  by generating successors of already-explored states

(a.k.a. expanding states)

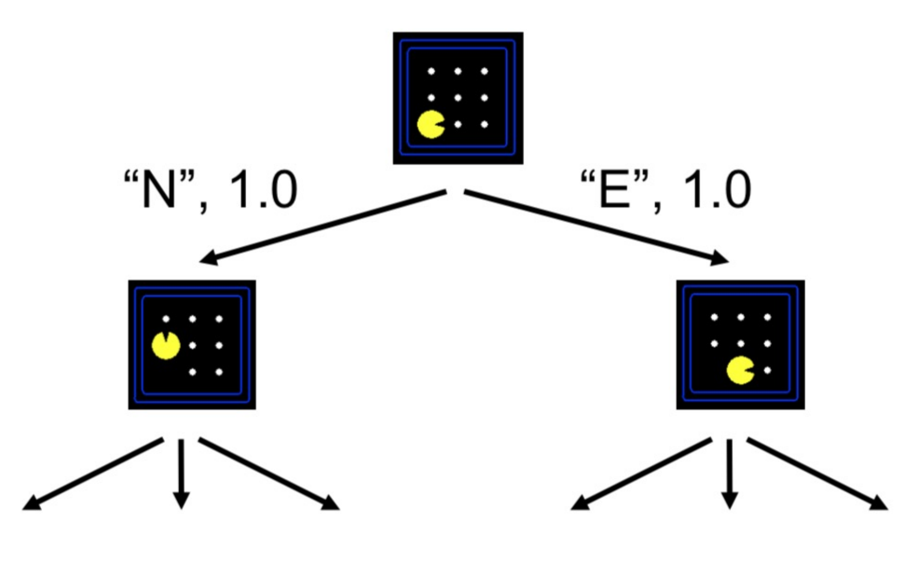
```
function TREE-SEARCH(problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```

Search Tree Example



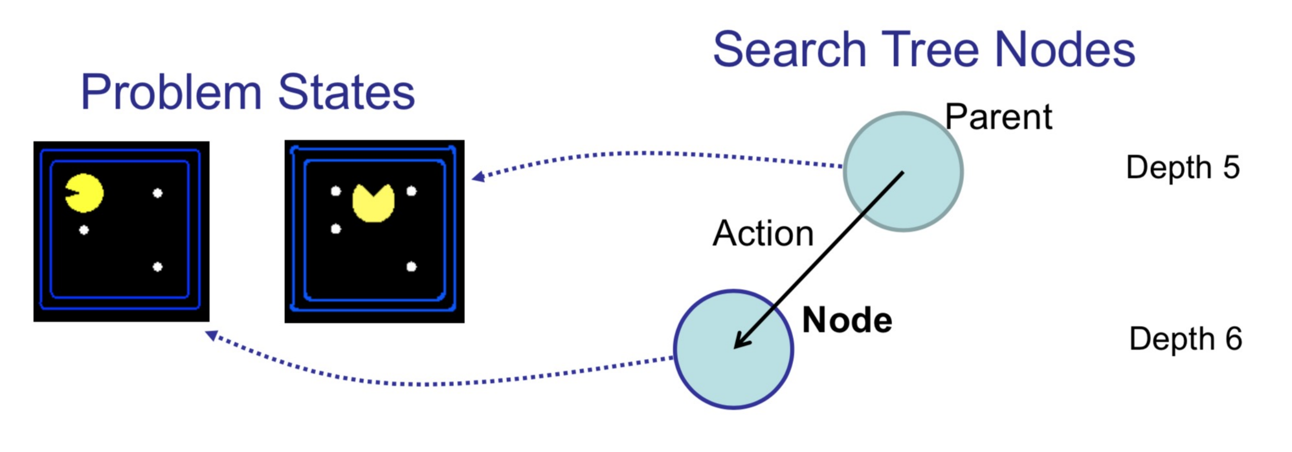
Search Tree

- A search tree:
  - Start state at the root node
  - Children correspond to successors
  - Nodes ***contain*** states, correspond to PLANS to those states
  - Edges are labeled with actions and costs
  - For most problems, we can never actually build the whole tree



States vs. Nodes

- Vertices in state space graphs are problem states
- Represent an abstracted state of the world
- Have successors, can be goal / non-goal, have multiple predecessors
- Vertices in search trees ("Nodes") are plans
- Contain a **problem state** and one parent, a path length, a depth, and a cost
- Represent a plan (sequence of actions) which results in the node's state
- **The same problem state may be achieved by multiple search tree nodes**



Search Strategies

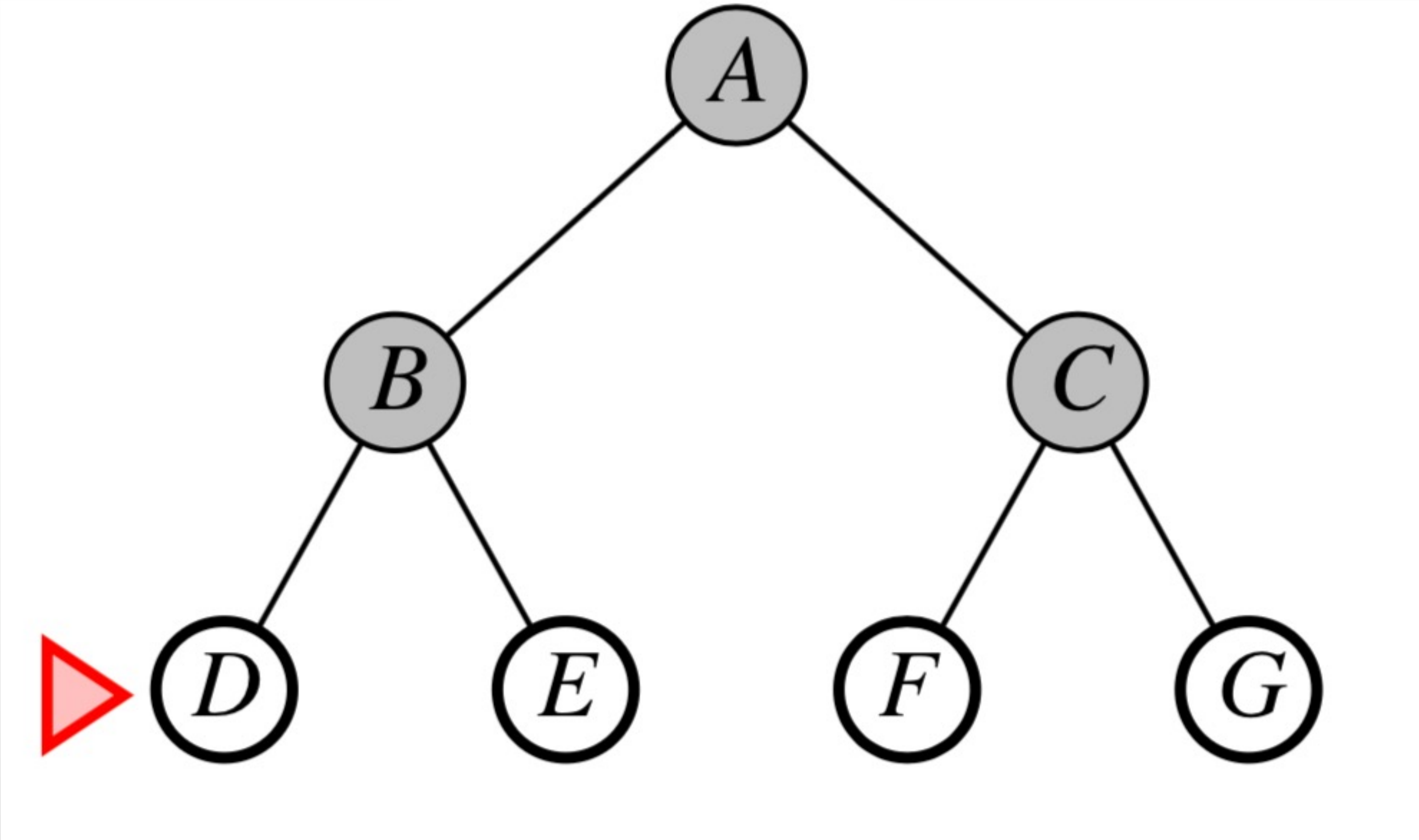
- A strategy is defined by picking **the order of node expansion**
- Strategies are evaluated along the following dimensions:
  - **completeness:** does it always find a solution if one exists?
  - **time complexity:** number of nodes generated/expanded
  - **space complexity:** maximum number of nodes in memory
  - **optimality:** does it always find a least-cost solution?
- Time and space complexity are measured in terms of
  - ***b***: maximum branching factor of the search tree
  - ***d***: depth of the least-cost solution
  - ***m***: maximum depth of the state space (may be ∞)

Search Strategies

- **Uninformed** strategies use only the information available in the problem definition
- Breadth-first search
- Uniform-cost search
- Depth-first search
- Depth-limited search
- Iterative deepening search

Breadth-first search

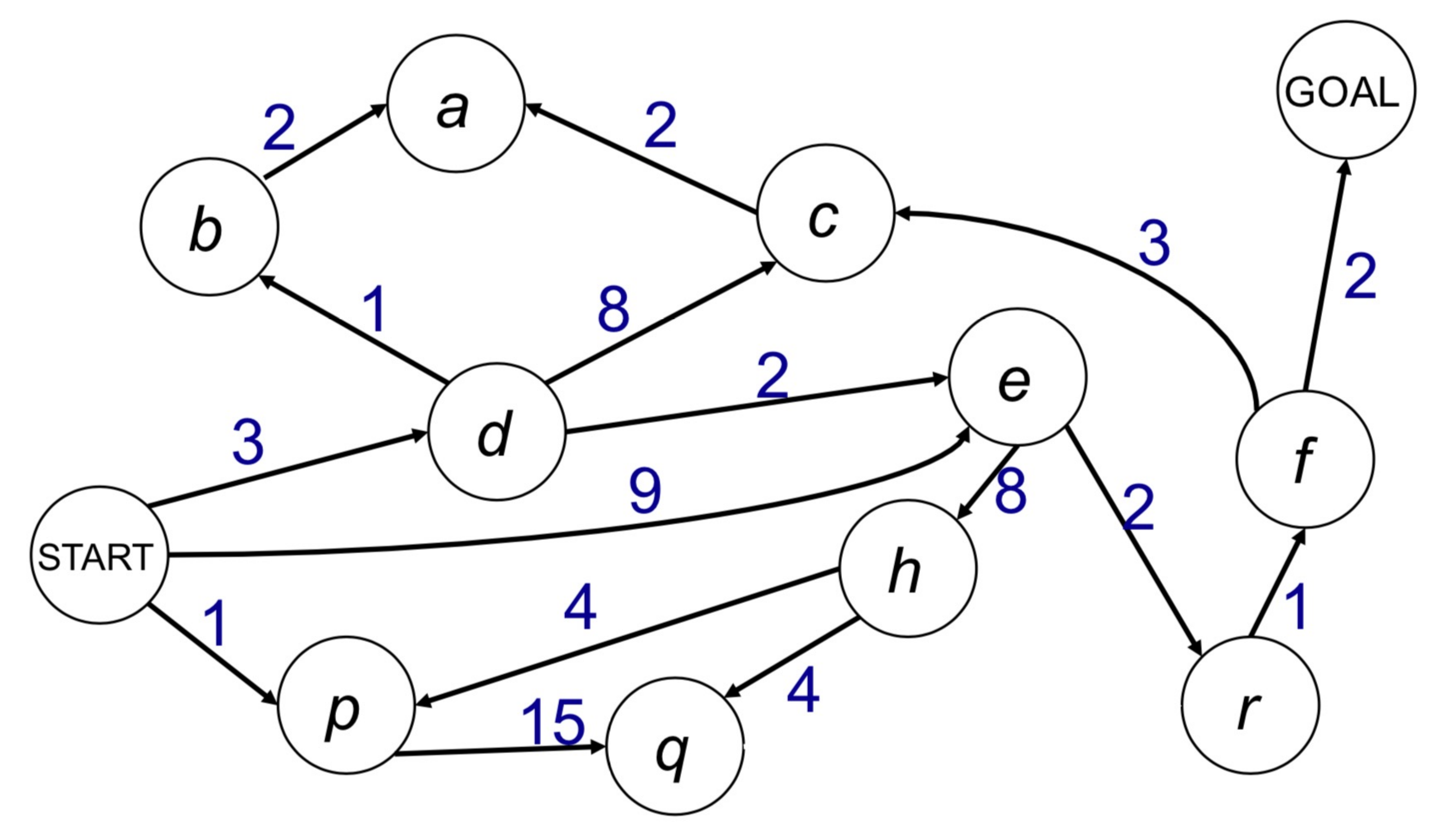
- Expand shallowest unexpanded node



Properties of breadth-first search

- **Complete**:
  - Yes (if b is finite)
- **Time**:
  - $1+b+b^2+b^3+\ldots+b^d+b(b^d-1)=O(b^{d+1})$, i.e. exp. in d
- **Space**:
  - $O(b^{d+1})$ (keeps every node in memory)
- **Optimal**:
  - Yes (if cost = 1 per step); not optimal in general
- Space is the big problem; can easily generate nodes at 100MB/sec so 24hrs = 8640GB.

Costs on Actions



- Objective: Path with smallest overall cost
- BFS will return shortest path in terms of number of transitions
  - It doesn't find the least cost path.

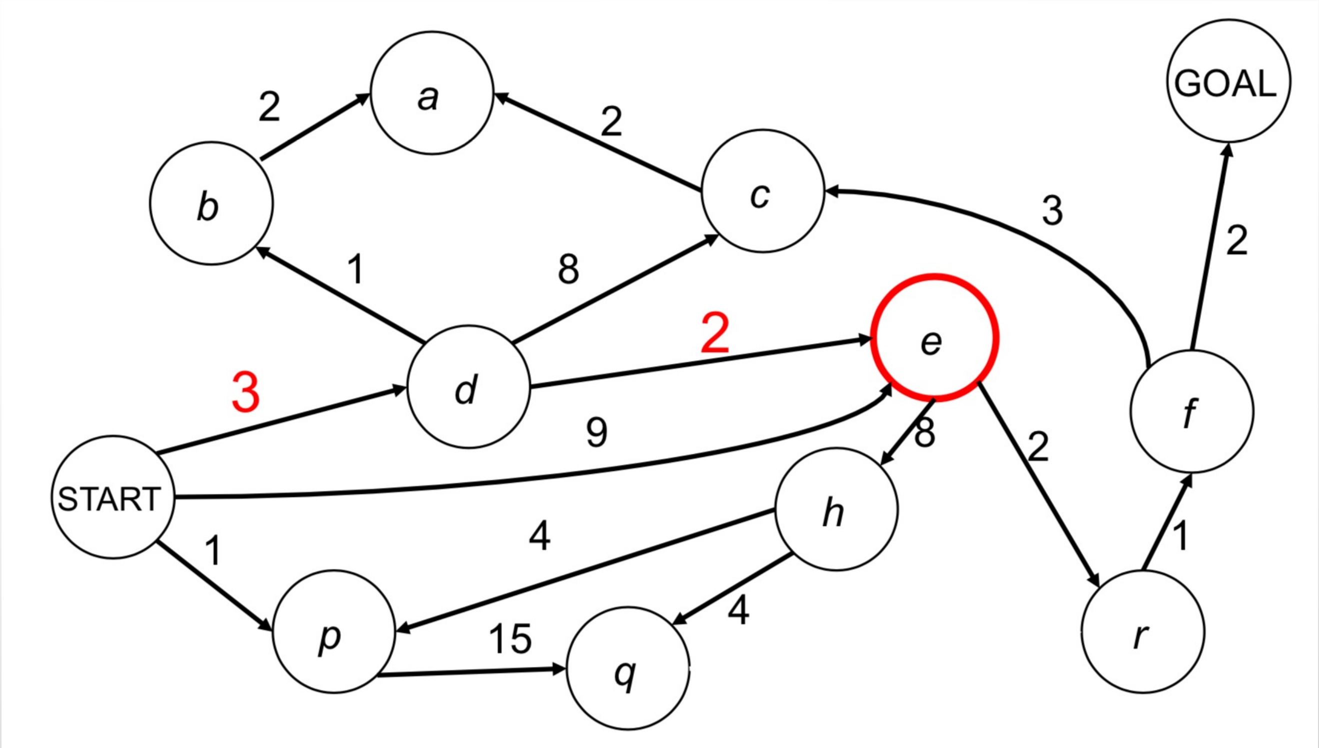
Best-first search

- Generalization of breadth-first search
- Cost function f(n) applied to each node
  - Breadth-first search : f(n) = depth(n)
  - Dijkstra’s Algorithm (Uniform cost) : f(n) = the sum of edge costs from start to n

Uniform Cost Search

- Best first, where

  f(n) = “cost from **start** to **n**”

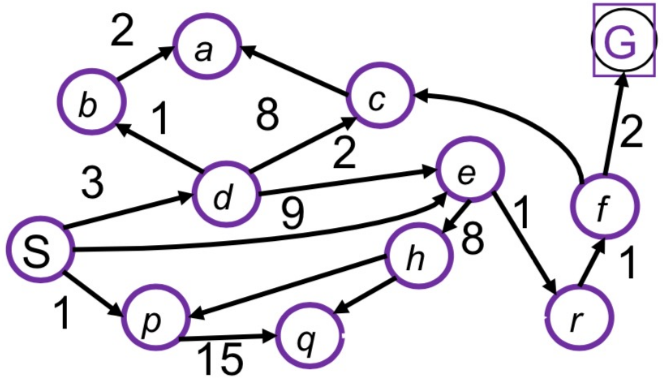


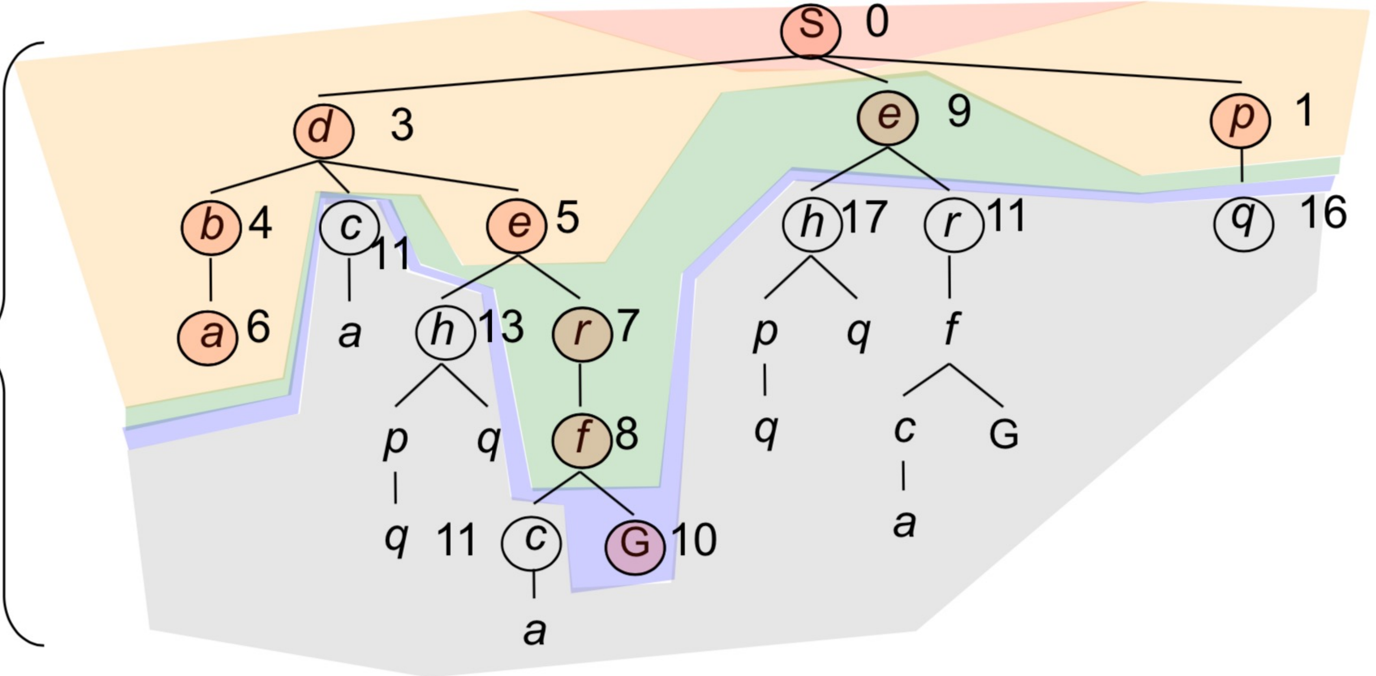
aka “Dijkstra’s Algorithm”

Uniform Cost Search (cont.)

Expansion order:

S, *p, d, b, e, a*, *r, f, e*, *G*



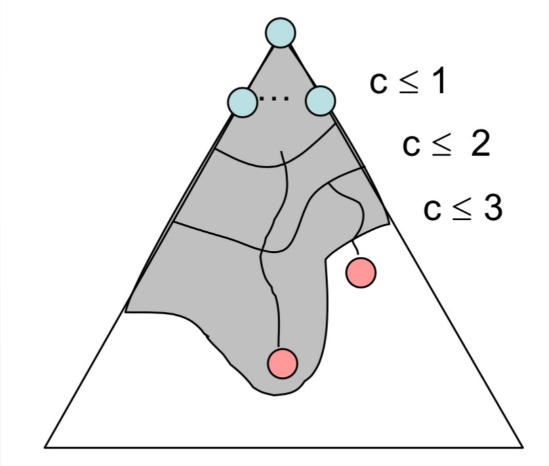


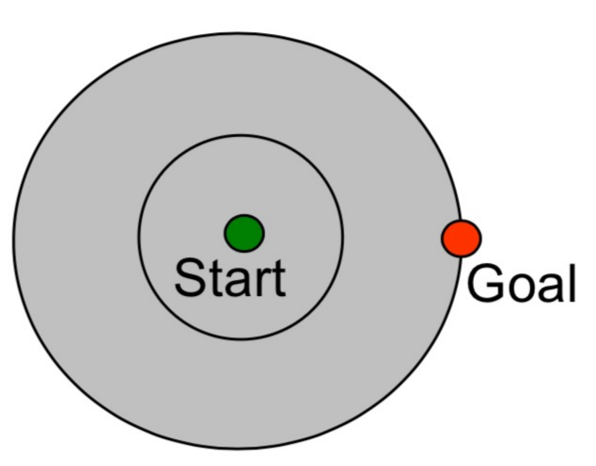
Cost contours (not all shown)

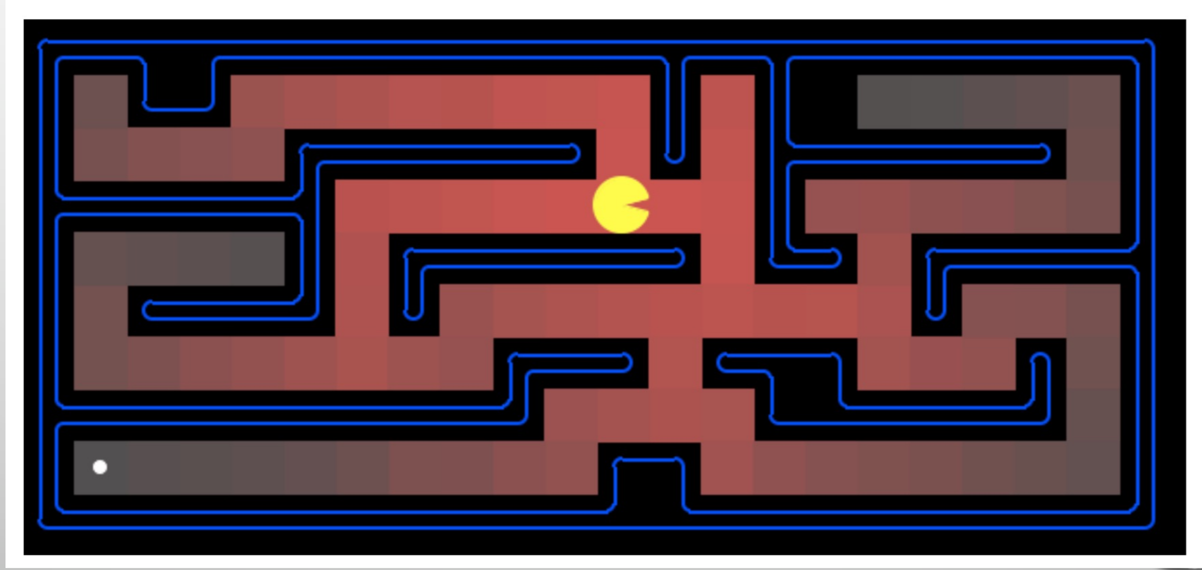
Uniform-cost search

- **Complete**:
  - Yes, if step cost $\geq \varepsilon$
- **Time**:
  - # of nodes with $f \leq$ cost of optimal solution, $O(b^{\lceil C*/\varepsilon \rceil})$ where C^* is the cost of the optimal solution
- **Space**:
  - # of nodes with $f \leq$ cost of optimal solution, $O(b^{\lceil C*/\varepsilon \rceil})$
- **Optimal**:
  - Yes—nodes expanded in increasing order of f(n)
- **Caveat**: Explores options in every "direction" (No information about goal location)

Uniform-cost search (cont.)

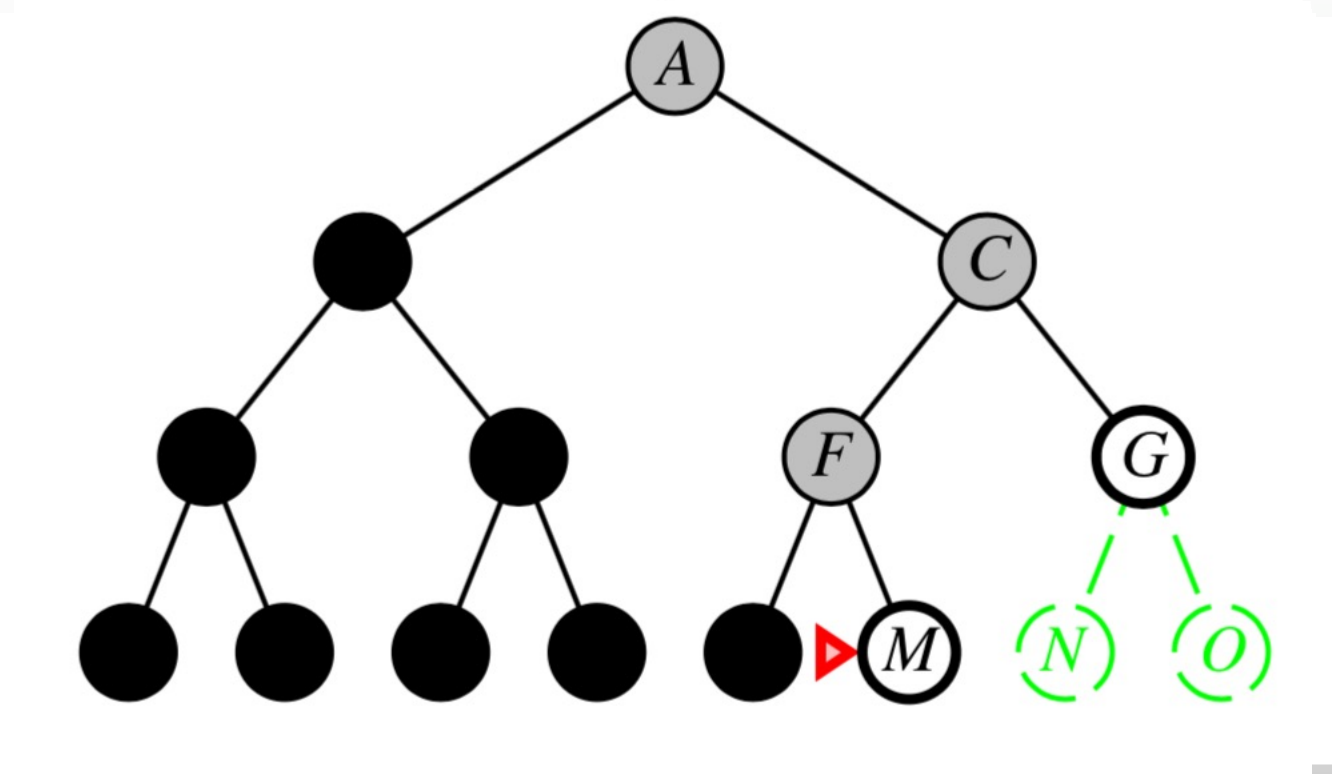






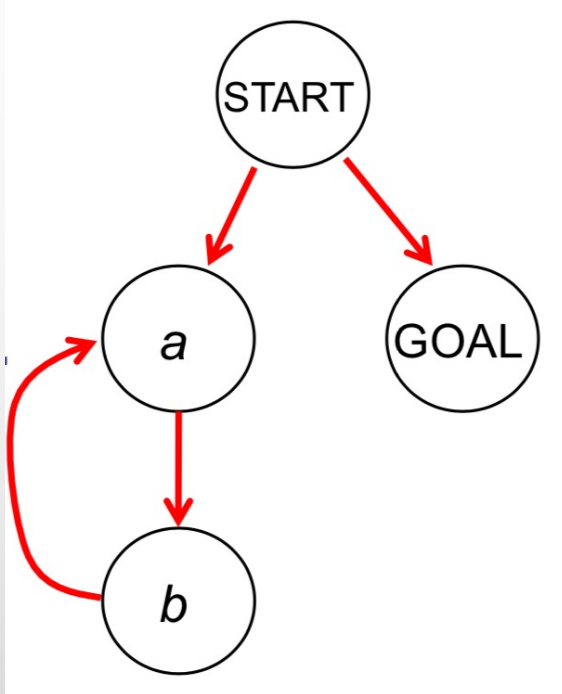
Depth-first search

- Expand deepest unexpanded node



Properties of depth-first search

- **Complete**:
  - No: fails in infinite-depth spaces, spaces with loops. Modify to avoid repeated states along path
  - ⇒ complete in finite spaces
- **Time**:
  - O(b^m): terrible if m is much larger than d
  - but if solutions are dense, may be much faster than breadth-first
- **Space**:
  - O(bm), i.e., linear space!
- **Optimal**:
  - No



Combining BFS and DFS?

- DFS is efficient in space complexity
- BFS is better in time complexity
- How can we combine strength of both in a method?

Depth-limited search

= depth-first search **with depth limit l**, i.e., nodes at depth l have no successors

```
function DEPTH-LIMITED-SEARCH( problem, limit) returns soln/fail/cutoff
   RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
   cutoff-occurred? ← false
   if GOAL-TEST(problem, STATE[node]) then return node
   else if DEPTH[node] = limit then return cutoff
   else for each successor in EXPAND(node, problem) do
        result ← RECURSIVE-DLS(successor, problem, limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result ≠ failure then return result
   if cutoff-occurred? then return cutoff else return failure
```

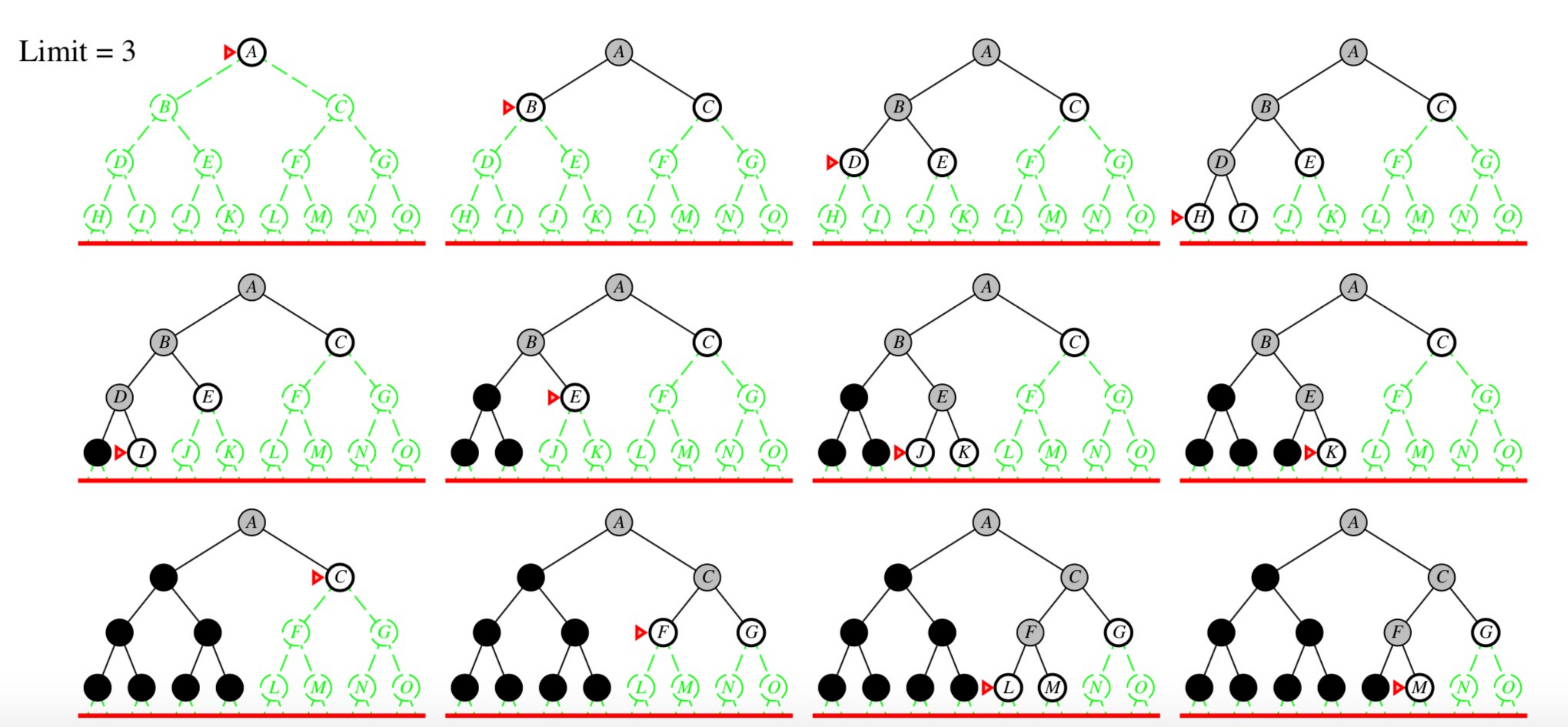
Iterative deepening search (cont.)

- Gradually increasing the limit in depth-limited search, until the solution is found:

```
function ITERATIVE-DEEPENING-SEARCH(problem) returns a solution
    inputs: problem, a problem

    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH(problem, depth)
        if result ≠ cutoff then return result
    end
```

Iterative deepening search (cont.)



Properties of iterative deepening search

- **Complete:**
  - Yes
- **Time:**
  - $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$
  - or more precisely $O(b^d(1 - 1/b)^{-2})$
- **Space:**
  - $O(bd)$
- **Optimal:**
  - Yes, if step cost = 1
  - Can be modified to explore uniform-cost tree

Properties of iterative deepening search (cont.)

- Numerical comparison for b = 10 and d = 5, solution at far right leaf:

  N(IDS) = 6+50+400+3,000+20,000+100,000=123,456

  N(BFS) = 10+100+1,000+10,000+100,000+999,990=1,111,100

- IDS does better because other nodes at depth d are not expanded
- BFS can be modified to apply goal test when a node is generated

Cost of iterative deepening

b	ratio ID to DFS
2	3
3	2
5	1.5
10	1.2
25	1.08
100	1.02

Speed on various benchmarks

	BFS Nodes	BFS Time		Iter. Deep. Nodes	Iter. Deep. Time
8 Puzzle	10^5	.01 sec		10^5	.01 sec
2x2x2 Rubik's	10^6	.2 sec		10^6	.2 sec
15 Puzzle	10^{13}	6 days	1Mx	10^{17}	20k yrs
3x3x3 Rubik's	10^{19}	68k yrs	8x	10^{20}	574k yrs
24 Puzzle	10^{25}	12B yrs		10^{37}	10^{23} yrs

Why the difference?

Rubik has higher branch factor
15 puzzle has greater depth

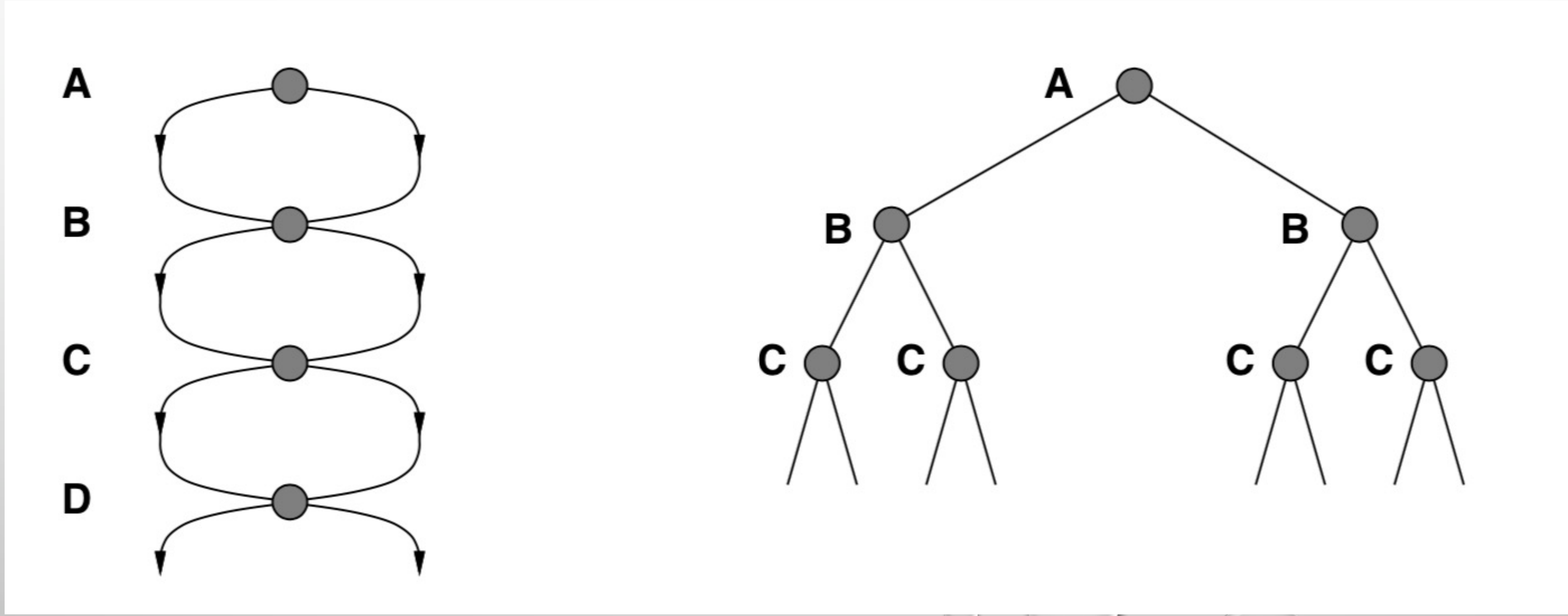
# of duplicates

Summary of algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes*	Yes*	No	Yes, if $l \geq d$	Yes
Time	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	b^m	b^l	b^d
Space	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	bm	bl	bd
Optimal?	Yes*	Yes	No	No	Yes*

Repeated states

- Failure to detect repeated states can turn a linear problem into an exponential one!



Graph Search

```
function GRAPH-SEARCH(problem, fringe) returns a solution, or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        if STATE[node] is not in closed then
            add STATE[node] to closed
            fringe ← INSERTALL(EXPAND(node, problem), fringe)
    end
```

Graph Search (cont.)

- On small problems
  - Graph search almost always better than tree search
- Implement your closed list as a dict. or set!
- On many real problems
  - Storage space is a huge concern.
  - Graph search impractical